

Cost of Capital with Levered Cost of Equity as the Risk of Tax Shields

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Abstract

We present the derivation of cost of capital under the assumption of risky tax shields discounted with the cost of levered equity. We show that the formulation is consistent and is derived from basic financial principles. This formulation is valid for finite cash flows and non growing perpetuities. In addition, it can be calculated without the circularity between value and discount rate.

Key Words

Cost of levered equity, firm valuation, tax shields, discount rate for tax shields, risk of tax shields.

JEL codes

M21, M40, M46, M41, G12, G31, J33

Introduction

Discounting tax shields, TS, is an issue that stimulates debate among academicians. The proposals range from the risk free cost of debt to the levered cost of equity. This raises several approaches to discounting tax shields.

The more popular and classical approach establishes that tax shields have to be discounted at the cost of debt, see for example, Modigliani and Miller (1958, 1963), Myers (1974), Lohman (1997), Brealey and Myers, 2003, and Damodaran (2002, 2005).

A second and more recent one says that the discount rate for tax shields should be the unlevered cost of equity, see for instance, Harris and Pringle, 1985, Ruback, 2002, Tham and Vélez-Pareja (2001, 2004, 2010).

Finally, Miles and Ezzell, 1985 and Arzac and Glosten, 2005, suggest discounting the tax shields at the cost of debt for year t and at the cost of unlevered equity for all subsequent years from $t+1$. For a discussion about this proposal see Vélez-Pareja, 2010.

On the other hand, Fernández, 2004, suggests that the value of tax shields is not the present value of tax shields and proposes a formulation independent from the cost of debt and tax shields. Cooper and Nyborg, 2006, reply that indeed the value of tax shields is the present value of tax shields.

Kolari, 2010, proposes that the risk of tax shields is the levered cost of equity. He derives a complex formulation for the levered cost of equity, K_e , under this assumption, as one root of a quadratic equation from where K_e is solved. Additionally, this solution has the traditional problem of circularity, because K_e depends on the market equity value.

In this work we propose a simple formulation for the levered cost of equity under the assumption that K_e as the risk of unlevered equity. This formulation has the virtue of no

circularity and works for finite cash flow and non growing perpetuities. Our solution is consistent with Kolari's proposal¹.

The work is organized in three sections. Section One presents the formulation for Ke. Section Two shows two examples where the formulation is verified for both, finite cash flows and non growing perpetuities. Section Three concludes. There is an Appendix where we show the algebraic derivation of this Ke.

Section One. Formulation for Ke.

Departing from basic relationships between value and cash flows we arrive to this simple formulation:

$$Ke_t = Ku_t + \frac{(Ku_t - Kd_t) \times D_{t-1}}{V_{t-1}^{Un} - D_{t-1}} \quad (1)$$

Where Ke is the levered cost of equity, Ku is the unlevered cost of equity, Kd is the cost of debt, V^{Un} is the firm unlevered value and D is the value of debt.

A question about (1) is if D could be greater than or equal to V^{Un} . If the firm is 100% leveraged, then it means that the owner is the debt holder and she will not receive more than the present value of the free cash flow, FCF. On the other hand, when D% is 100%, FCF is identical to the cash flow to debt, CFD and the firm will not get any TS (the firm defaults) and hence $D=V^{Un}$. This is a limiting case and when D is identical to V^{Un} , Ke is undetermined as it happens in other discount rate of TS frameworks. When the firm is 100% levered value of equity, E vanishes and Ke is undetermined.

Section Two. Example

First, we present a non growing perpetuity with constant debt.

¹ In private correspondence, Professor Kolari said: "Then compared your equation to my results and they are the same exactly." Message received Jul. 29, 2010 at 2:50 PM.

Table 1. Input data for non growing perpetuity and constant debt.

T	30%
Kd	6%
D	2
Ku	10%
FCF	1

Using (1) we calculate E as the value of cash flow to equity, CFE in perpetuity

Table 2. Calculation of Cash flows and values

$TS = K_dTD$	0.0360
$CFD = K_dD$	0.1200
$CFE = FCF + TS - CFD$	0.9160
$V^{Un} = FCF/K_u$	10.0000
$Ke_t = K_u + (K_u - K_d) \times D / (V^{Un} - D)$	11.00%
$VTS = TS/Ke$	0.3273
$V^L = APV = V^{Un} + VTS$	10.3273
$E = V^L - D$	8.3273
$E = CFE/Ke$	8.3273

The example for finite cash flows uses the same input data except for FCF. Again, using (1) we have:

Table 3. FCF and Calculation of CFE and Values

t		FCF ₁	FCF ₂	FCF ₃
FCF		1.0000	2.0000	3.0000
D	2.0000	2.0000	2.0000	
$TS_t = K_d D_{t-1} T$		0.0360	0.0360	0.0360
$CFD_t = K_d D_{t-1} T + D_t - D_{t-1}$		0.1200	0.1200	2.1200
$CFE_t = FCF_t + TS_t - CFD_t$		0.9160	1.9160	0.9160
$V^{Un} = PV(FCF \text{ at } K_u)$	4.8159	4.2975	2.7273	
$Ke_t = K_{u_t} + (K_{u_t} - K_{d_t}) \times D_{t-1} / (V^{Un}_{t-1} - D_{t-1})$		12.84%	13.48%	21.00%
$VTS = PV(TS \text{ at } Ke)$	0.0833	0.0579	0.0298	
$V = APV = V^{Un} + VTS$	4.8992	4.3555	2.7570	
$E = PV(CFE \text{ at } Ke)$	2.8992	2.3555	0.7570	
$V = E + D$	4.8992	4.3555	2.7570	

As seen in tables 2 and 3 the results are fully consistent. APV and present value of CFE plus debt match. Given the basic procedure to calculate Ke, we assert that this procedure and formulation are theoretically correct.

Concluding Remarks

We have shown a formulation for K_e when the risk of TS is K_e . The formula has two virtues: needs no circularity between value and discount rate and is valid for finite cash flows and non growing perpetuities. The results are consistent.

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Appendix

Formula for K_e when the risk of TS is K_e

$$\Psi_t \times V_{t-1}^{TS} = TS_t \quad (1)$$

$$K_e \times E_{t-1} = CFE_t \quad (2)$$

$$K_d \times D_{t-1} = CFD_t \quad (3)$$

$$K_u \times V_{t-1}^{Un} = FCF_t \quad (4)$$

$$V_{t-1}^L = V_{t-1}^{Un} + V_{t-1}^{TS} \quad (5a)$$

$$V_{t-1}^{Un} = E_{t-1} + D_{t-1} - V_{t-1}^{TS} \quad (5b)$$

$$FCF_t + TS_t = CFE_t + CFD_t \quad (6)$$

$$K_u \times V_{t-1}^{Un} + \psi_t \times V_{t-1}^{TS} = K_e \times E_{t-1} + K_d \times D_{t-1} \quad (7)$$

Assume that $\psi_t = K_e$

$$K_u \times V_{t-1}^{Un} + K_e \times V_{t-1}^{TS} = K_e \times E_{t-1} + K_d \times D_{t-1} \quad (8a)$$

$$K_e \times (E_{t-1} - V_{t-1}^{TS}) = K_u \times V_{t-1}^{Un} + K_d \times D_{t-1} \quad (8b)$$

$$K_e \times (V_{t-1}^{Un} - D_{t-1}) = K_u \times V_{t-1}^{Un} - K_u \times D_{t-1} - K_d \times D_{t-1} + K_u \times D_{t-1} \quad (8c)$$

$$K_e \times (V_{t-1}^{Un} - D_{t-1}) = K_u \times (V_{t-1}^{Un} - D_{t-1}) + (K_u - K_d) \times D_{t-1} \quad (8d)$$

$$K_e = K_u + \frac{(K_u - K_d) \times D_{t-1}}{V_{t-1}^{Un} - D_{t-1}} \quad (8e)$$