Cost of Equity and WACC for Perpetuities with Constant Growth

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Abstract

This paper shows a formulation for the cost of equity and the WACC for growing perpetuities. Some authors have derived the general expressions for those formulas but not specifically for perpetuities with constant growth. The result obtained is that a previously general formulation for a finite number of periods is also valid for growing perpetuities.

KEY WORDS

Firm valuation, cost of capital, cost of equity, perpetuities, constant growth, cash flow, free cash flow, WACC

JEL CLASSIFICATION

M21, M40, M46, M41, G12, G31, J33

Introduction

The discussion on the proper discount rate for FCF and CFE in a growing perpetuity has been systematically eluded in financial literature. In consequence, this paper studies the adequate discount rate for the cash flow to equity, CFE, and the free cash flow, FCF (this is, Ke and WACC respectively).

Literature review

Miles & Ezzel (1985) proposed a formulation for WACC in perpetuity assuming that during the first period the risk of the Tax Savings (TS) equals the cost of debt (Kd) and that it is equal to Ku (the cost of the unlevered equity) during the remaining periods. Taggart (1991) presented a formulation for the cost of equity Ke and the weighted average cost of capital WACC for non growing perpetuities. Fernández (2006 and 2007), in turn, proposes that WACC and Ke for growing perpetuities should be

$$\text{WACC} = \text{Ku}\left(1-\frac{V^{TS}}{V}\right)+\frac{gV^{TS}}{V} \quad (1)$$

and

$$\text{Ke} = \text{Ku} + \frac{D}{E}\left[\text{Ku}-\text{Kd}(1-T)\right]-\frac{V^{TS}}{E}(\text{Ku}-g) \quad (2)$$

Where D and E are the market values for Debt and Equity, V^{TS} is the present value of the Tax Savings, T is the Tax Rate, and g is the constant growth rate in perpetuity.

For him, Ke and WACC are independent from the discount rate for TS[1].

Miles and Ezzell propose

$$\text{WACC} = \text{Ku} - \frac{D\%\,\text{KdT}(1+\text{Ku})}{1+\text{Kd}} \quad (3)$$

$$\text{Ke} = \text{Ku} - \frac{D\%\,\text{KdT}(1+\text{Ku})}{1+\text{Kd}} \quad (4)$$

Where D% corresponds to the company leverage: $D\% = \frac{D}{D+E}$

[1] Fernández considers that the value of tax shields for growing perpetuities is VTS=KuDT/(Ku-g),

In consequence, for them Ke and WACC are also independent from the discount rate for TS. Tham and Vélez-Pareja (2002 and 2004) proposed the following for Ke and WACC:

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right)\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t\right)\frac{V_{t-1}^{TS}}{E_{t-1}} \qquad (5)$$

$$WACC_t = Ku_t - (Ku_t - \psi_t)\frac{V_{t-1}^{TS}}{V_{t-1}^{L}} - \frac{TS_t}{V_{t-1}^{L}} \qquad (6)$$

Vélez-Pareja (2010) analyzing the behavior of tax shields suggests an alternative to M&E's proposal and concludes that the discount rate for TS should be Ku.

Departing from basic relationships of cash flows and values such as

$$V_{t-1}^{Un} + V_{t-1}^{TS} = E_{t-1} + D_{t-1} \qquad (7)$$

$$FCF_t + TS_t = CFE_t + CFD_t \qquad (8)$$

$$V_{t-1}^{Un} = \frac{FCF_t}{Ku_t - g} \qquad (9)$$

$$E_{t-1} = \frac{CFE_t}{Ke_t - g} \qquad (10)$$

$$V_{t-1}^{TS} = \frac{TS_t}{\psi_t - g} \qquad (11)$$

$$D_{t-1} = \frac{CFD_t}{Kd_t - g} \qquad (12)$$

We arrive to the following expressions for WACC and Ke:

$$\mathbf{Ke_t = Ku_t + \left(Ku_t - Kd_t\right).\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t\right).\frac{V_{t-1}^{TS}}{E_{t-1}}} \qquad \mathbf{(13)}$$

$$\mathbf{WACC_t = Ku_t - (Ku_t - \psi_t).\frac{V_{t-1}^{TS}}{V_{t-1}^{L}} - \frac{TS_t}{V_{t-1}^{L}}} \qquad \mathbf{(14)}$$

These expressions are identical to those proposed for finite periods by Tham and Vélez-Pareja (2002 and 2004). See Appendix A for derivation.

When we assume $\psi = Ku$ we obtain

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right) . \frac{D_{t-1}}{E_{t-1}} \quad (15)$$

$$WACC_t = Ku_t - \frac{TS_t}{V^L_{t-1}} \quad (16)$$

When we assume ψ = Kd we obtain

$$Ke_t = Ku_t + \left(Ku_t - Kd_t\right) \left[\frac{D_{t-1}}{E_{t-1}} - \frac{V^{TS}_{t-1}}{E_{t-1}}\right] \quad (17)$$

$$WACC_t = Ku_t - (Ku_t - Kd_t) . \frac{V^{TS}_{t-1}}{V^L_{t-1}} - \frac{TS_t}{V^L_{t-1}} \quad (18)$$

These expressions depend indirectly on g, given that the values of E_{t-1} and $V^{TS}{}_{t-1}$ depend, in turn, on it.

Conclusion

In the present paper we have arrived to the conclusion that the formulas for the cost of equity (Ke) and WACC in the case of perpetuities with constant growth depend on the rate of discount for TS, and that the expressions derived are equal to those previously deduced by Tham and Vélez-Pareja for finite periods. This, in turn, should not come as a surprise because the basic relationships used by them in their derivation must hold true for any period, even in a perpetuity.

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Appendix A

$$V_{t-1}^{Un} + V_{t-1}^{TS} = E_{t-1} + D_{t-1} \qquad (1A)$$

$$FCF_t + TS_t = CFE_t + CFD_t \qquad (2A)$$

$$V_{t-1}^{Un} = \frac{FCF_t}{Ku_t - g} \qquad (3A)$$

$$E_{t-1} = \frac{CFE_t}{Ke_t - g} \qquad (4A)$$

$$D_{t-1} = \frac{CFD_t}{Kd_t - g} \qquad (5A)$$

$$V_{t-1}^{TS} = \frac{TS_t}{\psi_t - g} \qquad (6A)$$

$$\frac{FCF_t}{Ku_t - g} + \frac{TS_t}{\psi_t - g} = \frac{CFE_t}{Ke_t - g} + \frac{CFD_t}{Kd_t - g} \qquad (7A)$$

$$= (3A), (4A), (5A)\ y\ (6A)\ in\ (1A)$$

$$\frac{CFE_t}{Ke_t - g} = \frac{FCF_t}{Ku_t - g} + \frac{TS_t}{\psi_t - g} - \frac{CFD_t}{Kd_t - g} \qquad (8A)$$

$$\frac{CFE_t}{Ke_t - g} = \frac{FCF_t}{Ku_t - g} + \frac{TS_t}{\psi_t - g} - \frac{FCF_t + TS_t - CFE_t}{Kd_t - g} \qquad (9A)$$

$$= (2A)\ in\ (8A)$$

$$\frac{CFE_t}{Ke_t - g} = \frac{FCF_t.(\psi_t - g).(Kd_t - g) + TS_t.(Ku_t - g).(Kd_t - g) - (FCF_t + TS_t - CFE_t).(Ku_t - g).(\psi_t - g)}{(Ku_t - g).(\psi_t - g).(Kd_t - g)}$$

$$\frac{CFE_t}{Ke_t - g} = \frac{FCF_t.(\psi_t - g).(Kd_t - g - Ku_t + g) + TS_t.(Ku_t - g).(Kd_t - g - \psi_t + g) + CFE_t.(Ku_t - g).(\psi_t - g)}{(Ku_t - g).(\psi_t - g).(Kd_t - g)}$$

$$\frac{CFE_t}{Ke_t - g} = \frac{FCF_t.(\psi_t - g).(Kd_t - Ku_t) + TS_t.(Ku_t - g).(Kd_t - \psi_t) + CFE_t.(Ku_t - g).(\psi_t - g)}{(Ku_t - g).(\psi_t - g).(Kd_t - g)}$$

$$\frac{CFE_t}{Ke_t - g} = \frac{FCF_t.(Kd_t - Ku_t)}{(Ku_t - g).(Kd_t - g)} + \frac{TS_t.(Kd_t - \psi_t)}{(\psi_t - g).(Kd_t - g)} + \frac{CFE_t}{(Kd_t - g)} \qquad (10A)$$

$$\frac{CFE_t}{Ke_t - g} = \frac{V_{t-1}^{Un}.(Kd_t - Ku_t)}{(Kd_t - g)} + \frac{V_{t-1}^{TS}.\left(Kd_t - \psi_t\right)}{(Kd_t - g)} + \frac{CFE_t}{(Kd_t - g)} \qquad (11A)$$

$$= (3A)\ y\ (6A)\ en\ (10A)$$

$$\frac{CFE_t}{Ke_t - g} = \frac{V_{t-1}^{Un}.(Kd_t - Ku_t) + V_{t-1}^{TS}.\left(Kd_t - \psi_t\right) + CFE_t}{(Kd_t - g)}$$

$$Ke_t - g = \frac{CFE_t.(Kd_t - g)}{V_{t-1}^{Un}.(Kd_t - Ku_t) + V_{t-1}^{TS}.\left(Kd_t - \psi_t\right) + CFE_t} \qquad (12A)$$

$$Ke_t - g = \frac{E_{t-1}.(Ke_t - g).(Kd_t - g)}{V_{t-1}^{Un}.(Kd_t - Ku_t) + V_{t-1}^{TS}.\left(Kd_t - \psi_t\right) + E_{t-1}.(Ke_t - g)} \qquad (13A)$$

$$= (4A)\ en\ (12A)$$

$$E_{t-1}.(Kd_t - g) = V_{t-1}^{Un}.(Kd_t - Ku_t) + V_{t-1}^{TS}.\left(Kd_t - \psi_t\right) + E_{t-1}.(Ke_t - g)$$

$$E_{t-1}.Ke_t - E_{t-1}.g = E_{t-1}.(Kd_t - g) - V_{t-1}^{Un}.(Kd_t - Ku_t) - V_{t-1}^{TS}.\left(Kd_t - \psi_t\right)$$

$$E_{t-1}.Ke_t = E_{t-1}.Kd_t - V_{t-1}^{Un}.(Kd_t - Ku_t) - V_{t-1}^{TS}.\left(Kd_t - \psi_t\right)$$

$$Ke_t = Kd_t + (Ku_t - Kd_t).\frac{V_{t-1}^{Un}}{E_{t-1}} - \left(Kd_t - \psi_t\right).\frac{V_{t-1}^{TS}}{E_{t-1}} \qquad (14A)$$

$$Ke_t = Kd_t + (Ku_t - Kd_t).\frac{E_{t-1} + D_{t-1} - V_{t-1}^{TS}}{E_{t-1}} - \left(Kd_t - \psi_t\right).\frac{V_{t-1}^{TS}}{E_{t-1}} \qquad (15A)$$

$$= (1A)\ in\ (14A)$$

$$Ke_t = Kd_t + (Ku_t - Kd_t) + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - (Ku_t - Kd_t).\frac{V_{t-1}^{TS}}{E_{t-1}} - \left(Kd_t - \psi_t\right).\frac{V_{t-1}^{TS}}{E_{t-1}}$$

$$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - Kd_t + Kd_t - \psi_t\right).\frac{V_{t-1}^{TS}}{E_{t-1}}$$

$$\boldsymbol{Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t\right).\frac{V_{t-1}^{TS}}{E_{t-1}}} \qquad \mathbf{(16A)}$$

$$FCF_t = V_{t-1}^{Un}.(Ku_t - g) \qquad (17A)$$

$$FCF_t = V_{t-1}^{L}.(WACC_t - g) \qquad (18A)$$

$$V_{t-1}^{TS}.\left(\psi_t - g\right) = TS_t \qquad (6A)$$

$$V_{t-1}^{TS}.\psi_t - V_{t-1}^{TS}.g = TS_t$$

$$g = \frac{V_{t-1}^{TS}.\,\psi_t - TS_t}{V_{t-1}^{TS}} \qquad (19A)$$

$$\left(V_{t-1}^{L} - V_{t-1}^{TS}\right).\,(Ku_t - g) = V_{t-1}^{L}.\,(WACC_t - g) \qquad (17A) = (18A)$$

$$V_{t-1}^{L}.\,Ku_t - V_{t-1}^{TS}.\,Ku_t - V_{t-1}^{L}.\,g + V_{t-1}^{TS}.\,g = V_{t-1}^{L}.\,WACC_t - V_{t-1}^{L}.\,g$$

$$V_{t-1}^{L}.\,(Ku_t - g + g) - V_{t-1}^{TS}.\,(Ku_t - g) = V_{t-1}^{L}.\,WACC_t$$

$$V_{t-1}^{L}.\,Ku_t - V_{t-1}^{TS}.\,(Ku_t - g) = V_{t-1}^{L}.\,WACC_t$$

$$WACC_t = Ku_t - \frac{V_{t-1}^{TS}.\,(Ku_t - g)}{V_{t-1}^{L}}$$

$$WACC_t = Ku_t - \frac{V_{t-1}^{TS}.\,Ku_t}{V_{t-1}^{L}} + \frac{V_{t-1}^{TS}.\,g}{V_{t-1}^{L}} \qquad (20A)$$

$$WACC_t = Ku_t - \frac{V_{t-1}^{TS}.\,Ku_t}{V_{t-1}^{L}} + \frac{V_{t-1}^{TS}.\,(V_{t-1}^{TS}.\,\psi_t - TS_t)}{V_{t-1}^{L}.\,V_{t-1}^{TS}} \qquad (21A)$$

$$= (19A)\ in\ (20A)$$

$$WACC_t = Ku_t - \frac{V_{t-1}^{TS}.\,Ku_t}{V_{t-1}^{L}} + \frac{V_{t-1}^{TS}.\,\psi_t - TS_t}{V_{t-1}^{L}}$$

$$WACC_t = Ku_t - \frac{V_{t-1}^{TS}.\,Ku_t - V_{t-1}^{TS}.\,\psi_t + TS_t}{V_{t-1}^{L}}$$

$$WACC_t = Ku_t - \frac{V_{t-1}^{TS}.\,(Ku_t - \psi_t) + TS_t}{V_{t-1}^{L}}$$

$$\boldsymbol{WACC_t = Ku_t - (Ku_t - \psi_t).\frac{V_{t-1}^{TS}}{V_{t-1}^{L}} - \frac{TS_t}{V_{t-1}^{L}}} \qquad (\mathbf{22A})$$