Cost of Capital, Cost of Equity and Value without Circularity

for Constant Growth Perpetuities

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First Version: 28 Jun. 2010
This Version: 15 Aug. 2010

Abstract

This paper presents a formulation for the cost of equity, the WACC, and the value of equity and firm for growing perpetuities without circularity, using some previous results. We also derive a relationship between leverage, D%, and growth, given a value of debt at instant zero.

KEY WORDS

Firm valuation, cost of capital, cost of equity, cash flows, free cash flow, cash flow to equity, capital cash flow, WACC, perpetuities, constant growth, circularity

JEL CLASSIFICATION

M21, M40, M46, M41, G12, G31, J33

Introduction

Mejía and Vélez-Pareja (2010a) presented the derivation of a set of formulas for avoiding circularity in the calculation of discount rate and value for finite cash flows. This paper presents the general formulation for perpetuities with constant growth. From these formulas we have the case for non growing perpetuities for different scenarios depending on the discount rate of tax shields.

Formulas without Circularity

In deriving the non circularity formulas we assume the following:

1. For terminal value of a forecasted finite cash flow we know debt at year N, the last forecasted year (which in the derived formulae corresponds to year t-1), which results from the financial planning model at N.
2. Growth g is defined a priori based on the expectations and goals of the firm. Debt grows according to g and is readjusted every year.
3. The firm leverage, D%, is constant and is a function of g. In addition, D% is "internally" defined in the model, and the result (in case of numerically checking the perpetuity results) should be a constant leverage which indicates consistency in the valuation. The derivation of D% as a function of g is shown below.

Mejía and Vélez-Pareja (2010b) presented the proper formulation for Ke and WACC, for growing perpetuities, as follows:

$$\mathbf{Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t\right).\frac{V^{TS}_{t-1}}{E_{t-1}}} \qquad (1)$$

$$\mathbf{WACC_t = Ku_t - (Ku_t - \psi_t).\frac{V^{TS}_{t-1}}{V^{L}_{t-1}} - \frac{TS_t}{V^{L}_{t-1}}} \qquad (2)$$

Nevertheless, the formulations shown above imply circularity in their structure. In order to solve this issue, we derive departing from them the formulations for Ke, WACC, equity and firm market values without circularity with the following results:

$$\mathbf{Ke_t = \frac{Ku_t.\,CFE_t - g.\left[(Ku_t - Kd_t).\,D_{t-1} - (Ku_t - \psi_t).\,V^{TS}_{t-1}\right]}{CFE_t - (Ku_t - Kd_t).\,D_{t-1} + (Ku_t - \psi_t).\,V^{TS}_{t-1}}} \quad (3)$$

$$\mathbf{E_{t-1} = \frac{CFE_t - (Ku_t - Kd_t).\,D_{t-1} + (Ku_t - \psi_t).\,V^{TS}_{t-1}}{Ku_t - g}} \quad (4)$$

$$\mathbf{WACC_t = \frac{Ku_t.\,FCF_t + g.\,[(Ku_t - \psi_t).\,V^{TS}_{t-1} + TS_t]}{FCF_t + (Ku_t - \psi_t).\,V^{TS}_{t-1} + TS_t}} \quad (5)$$

$$\mathbf{E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - \psi_t).\,V^{TS}_{t-1} + TS_t}{Ku_t - g}} \quad (6)$$

Complete derivation can be found in Appendix A. We can see how these formulas look like, for selected values of ψ (the discount rate for the tax shields), and for two scenarios of growth (g=0 and g>0):

Table 1. Formulas for selected values of ψ and for g=0 and g>0

	ψ = Ku	**ψ = Kd**
g > 0		
Ke	$Ke_t = \frac{Ku_t.CFE_t - g.[(Ku_t - Kd_t).D_{t-1}]}{CFE_t - (Ku_t - Kd_t).D_{t-1}}$ (7)	$Ke_t = \frac{Ku_t.CFE_t - g.[(Ku_t - Kd_t).(D_{t-1} - V^{TS}_{t-1})]}{CFE_t - (Ku_t - Kd_t).(D_{t-1} - V^{TS}_{t-1})}$ (11)
E	$E_{t-1} = \frac{CFE_t - (Ku_t - Kd_t).D_{t-1}}{Ku_t - g}$ (8)	$E_{t-1} = \frac{CFE_t - (Ku_t - Kd_t).(D_{t-1} - V^{TS}_{t-1})}{Ku_t - g}$ (12)
WACC	$WACC_t = \frac{Ku_t.FCF_t + g.TS_t}{FCF_t + TS_t}$ (9)	$WACC_t = \frac{Ku_t.FCF_t + g.[(Ku_t - Kd_t).V^{TS}_{t-1} + TS_t]}{FCF_t + (Ku_t - Kd_t).V^{TS}_{t-1} + TS_t}$ (13)
V	$E_{t-1} + D_{t-1} = \frac{FCF_t + TS_t}{Ku_t - g}$ (10)	$E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - Kd_t).V^{TS}_{t-1} + TS_t}{Ku_t - g}$ (14)
g = 0		
Ke	$Ke_t = \frac{Ku_t.CFE_t}{CFE_t - (Ku_t - Kd_t).D_{t-1}}$ (15)	$Ke_t = \frac{Ku_t.CFE_t}{CFE_t - (Ku_t - Kd_t).(D_{t-1} - V^{TS}_{t-1})}$ (19)
E	$E_{t-1} = \frac{CFE_t - (Ku_t - Kd_t).D_{t-1}}{Ku_t}$ (16)	$E_{t-1} = \frac{CFE_t - (Ku_t - Kd_t).(D_{t-1} - V^{TS}_{t-1})}{Ku_t}$ (20)
WACC	$WACC_t = \frac{Ku_t.FCF_t}{FCF_t + TS_t}$ (17)	$WACC_t = \frac{Ku_t.FCF_t}{FCF_t + (Ku_t - Kd).V^{TS}_{t-1} + TS_t}$ (21)
V	$E_{t-1} + D_{t-1} = \frac{FCF_t + TS_t}{Ku_t}$ (18)	$E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - Kd_t).V^{TS}_{t-1} + TS_t}{Ku_t}$ (22)

Observe that value of firm, when ψ=Ku and g>0 and when ψ=Ku and g=0, is equivalent to discounting the Capital Cash Flow with Ku which causes circularity to disappear, as Tham and Vélez-Pareja (2002 and 2004) report.

Since previous formulas have been derived from (1) and (2), the general expressions for Ke and WACC, these results are coherent and consistent with the well-known textbook formulas and results. These formulas are derived back and shown in Appendix B.

The Appendices B and C show how other well-known results can be deduced as particular cases from the formulas proposed in this section. In the next table we summarize the results.

Table 2. Traditional textbook formulas for selected values of ψ and for g=0 and g>0

	ψ = Ku	**ψ = Kd**
g = 0		
Ke	$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}}$ (23)	$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}}.(1-T)$ (24)
WACC	$WACC_t = \frac{Ke_t.E_{t-1}+Kd_t.D_{t-1}.(1-T)}{E_{t-1}+D_{t-1}}$ (25)	
V		$E_{t-1} + D_{t-1} = V^u_{t-1} + D_{t-1}.T$ (26)
g > 0		
WACC	$WACC_t = \frac{Ke_t.E_{t-1}+Kd_t.D_{t-1}.(1-T)}{E_{t-1}+D_{t-1}}$ (27)	
V		$E_{t-1} + D_{t-1} = V^u_{t-1} + \frac{D_{t-1}.T.Ku_t}{Ku_t - g}$ (28)

The value of the Levered Firm as a non-growing perpetuity according to Modigliani & Miller (1963) and Myers (1974) is equation (26). Analogously, the formula for the value of the levered firm as a growing perpetuity proposed by Modigliani & Miller (1963) and Lewellen and Emery (1986) is (28).

Leverage and Growth

Growth g is not independent from leverage D%, and vice versa. When we calculate the perpetuity (in particular, when we use the perpetuity as a terminal or continuing value), we set a constant leverage that is introduced in the formulation of Ke and WACC. Debt at time zero (N in the case of terminal or continuing value and t-1 in the herein derived formulas), should be consistent with the terminal value and the leverage in perpetuity. In the case of a perpetuity starting at zero, debt may be defined as a percentage of value, whereas in the instance of a terminal value, debt at N is a result provided by the financial planning model. This means that in the latter case we have to adjust debt at N to the value required by the terminal value and the leverage.

The alternative is to define growth consistently with the leverage implied by debt at N. What we do is precisely define D% as a function of g (or conversely, g as a function of D%). We consider that management should define growth goals and hence, D% should be defined as we do, in terms of g.

Following the previous line of thought, we derived the relationship that must hold between the leverage, the initial amount of debt and the growth rate, in Appendix D as follows:

$$\mathbf{D\%} = \frac{\mathbf{D_{t-1}.(Ku_t - g).(\psi_t - g)}}{\mathbf{FCF_t.(\psi_t - g) + D_{t-1}.Kd_t.T.(Ku_t - g)}} \qquad \mathbf{(D11)}$$

Conclusions

We have derived a set of non-circular formulas for perpetuities with constant growth and showed that some previously well-know results can be viewed as particular cases of the general expressions proposed here. In addition, we have shown that there exists a precise relationship that must hold between the growth rate, the initial level of debt, and the leverage (when the latter is set as a target level), in order to preserve the consistency of the calculations.

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Appendix A

General Non-Circularity Formulation for Ke with Constant Growth.

From (1) we have

$$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t\right).\frac{V^{TS}_{t-1}}{E_{t-1}} \quad (A1)$$

$$E_{t-1} = \frac{CFE_t}{Ke_t - g} \quad (A2)$$

$$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}.(Ke_t - g)}{CFE_t} - \left(Ku_t - \psi_t\right).\frac{V^{TS}_{t-1}.(Ke_t - g)}{CFE_t} \quad (A3) = (A2) \text{ in } (A1)$$

$$Ke_t = \frac{Ku_t.CFE_t + (Ku_t - Kd_t).D_{t-1}.(Ke_t - g) - \left(Ku_t - \psi_t\right).V^{TS}_{t-1}.(Ke_t - g)}{CFE_t} \quad (A4a)$$

$$Ke_t.CFE_t = Ku_t.CFE_t + (Ku_t - Kd_t).D_{t-1}.(Ke_t - g) - \left(Ku_t - \psi_t\right).V^{TS}_{t-1}.(Ke_t - g) \quad (A4b)$$

$$Ke_t.[CFE_t - (Ku_t - Kd_t).D_{t-1} + (Ku_t - \psi_t).V^{TS}_{t-1}] = Ku_t.CFE_t - g.\left[(Ku_t - Kd_t).D_{t-1} - (Ku_t - \psi_t).V^{TS}_{t-1}\right] \quad (A4c)$$

$$\mathbf{Ke_t} = \frac{\mathbf{Ku_t.CFE_t - g.\left[(Ku_t - Kd_t).D_{t-1} - (Ku_t - \psi_t).V^{TS}_{t-1}\right]}}{\mathbf{CFE_t - (Ku_t - Kd_t).D_{t-1} + (Ku_t - \psi_t).V^{TS}_{t-1}}} \quad (\mathbf{A4d})$$

General Non-Circularity Formulation for Equity Market Value with Constant Growth.

From (A1) and (A2) we have

$$E_{t-1} = \frac{CFE_t}{Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t\right).\frac{V^{TS}_{t-1}}{E_{t-1}} - g} \quad (A5) = (A1) \text{ in } (A2)$$

$$E_{t-1} = \frac{CFE_t.E_{t-1}}{Ku_t.E_{t-1} + (Ku_t - Kd_t).D_{t-1} - \left(Ku_t - \psi_t\right).V^{TS}_{t-1} - g.E_{t-1}} \quad (A6a)$$

$$Ku_t.E_{t-1} + (Ku_t - Kd_t).D_{t-1} - \left(Ku_t - \psi_t\right).V^{TS}_{t-1} - g.E_{t-1} = CFE_t \quad (A6b)$$

$$E_{t-1}.(Ku_t - g) = CFE_t - (Ku_t - Kd_t).D_{t-1} + \left(Ku_t - \psi_t\right).V^{TS}_{t-1} \quad (A6c)$$

$$\mathbf{E_{t-1}} = \frac{\mathbf{CFE_t - (Ku_t - Kd_t).D_{t-1} + (Ku_t - \psi_t).V^{TS}_{t-1}}}{\mathbf{Ku_t - g}} \quad (\mathbf{A6d})$$

General Non-Circularity Formulation for WACC with Constant Growth.

From (2) we have

$$\mathrm{WACC_t = Ku_t - (Ku_t - \psi_t).\frac{V^{TS}_{t-1}}{E_{t-1} + D_{t-1}} - \frac{TS_t}{E_{t-1} + D_{t-1}}} \quad \text{(A7)}$$

$$\mathrm{E_{t-1} + D_{t-1} = \frac{FCF_t}{WACC_t - g}} \quad \text{(A8)}$$

$$\mathrm{WACC_t = Ku_t - (Ku_t - \psi_t).\frac{V^{TS}_{t-1}.(WACC_t - g)}{FCF_t} - \frac{TS_t.(WACC_t - g)}{FCF_t}} \quad \text{(A9) = (A8) in (A7)}$$

Solving for WACC

$$\mathrm{WACC_t = \frac{Ku_t.FCF_t - (Ku_t - \psi_t).V^{TS}_{t-1}.(WACC_t - g) - TS_t.(WACC_t - g)}{FCF_t}} \quad \text{(A10a)}$$

$$\mathrm{WACC_t.FCF_t = Ku_t.FCF_t - (Ku_t - \psi_t).V^{TS}_{t-1}.(WACC_t - g) - TS_t.(WACC_t - g)} \quad \text{(A10b)}$$

$$\mathrm{WACC_t.[FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t] = Ku_t.FCF_t + g.[(Ku_t - \psi_t).V^{TS}_{t-1} + TS_t]} \quad \text{(A10c)}$$

$$\mathbf{WACC_t = \frac{Ku_t.FCF_t + g.[(Ku_t - \psi_t).V^{TS}_{t-1} + TS_t]}{FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t}} \quad \textbf{(A10d)}$$

General Non-Circularity Formulation for Firm Value with Constant Growth.

$$\mathrm{E_{t-1} + D_{t-1} = \frac{FCF_t}{Ku_t - (Ku_t - \psi_t).\frac{V^{TS}_{t-1}}{E_{t-1} + D_{t-1}} - \frac{TS_t}{E_{t-1} + D_{t-1}} - g}} \quad \text{(A11) = (A7) in (A8)}$$

$$\mathrm{E_{t-1} + D_{t-1} = \frac{FCF_t.(E_{t-1} + D_{t-1})}{Ku_t.(E_{t-1} + D_{t-1}) - (Ku_t - \psi_t).V^{TS}_{t-1} - TS_t - g.(E_{t-1} + D_{t-1})}} \quad \text{(A12a)}$$

$$\mathrm{Ku_t.(E_{t-1} + D_{t-1}) - (Ku_t - \psi_t).V^{TS}_{t-1} - TS_t - g.(E_{t-1} + D_{t-1}) = FCF_t} \quad \text{(A12b)}$$

$$\mathrm{(Ku_t - g).(E_{t-1} + D_{t-1}) = FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t} \quad \text{(A12c)}$$

$$\mathbf{E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t}{Ku_t - g}} \quad \textbf{(A12d)}$$

Appendix B

1. **Particular Case for Ke if $g=0$, $\psi_t=Kd_t$, and $V^{TS}_{t-1}=(D_{t-1}.\ Kd_t\ .T)/\ Kd_t= D_{t-1}.T$**

$$Ke_t = \frac{Ku_t.CFE_t - g.\left[(Ku_t - Kd_t).D_{t-1} - \left(Ku_t - \psi_t\right).V_{t-1}^{TS}\right]}{CFE_t - (Ku_t - Kd_t).D_{t-1} + \left(Ku_t - \psi_t\right).V_{t-1}^{TS}} \quad \text{(A4d)}$$

$$Ke_t = \frac{Ku_t.CFE_t}{CFE_t - (Ku_t - Kd_t).D_{t-1} + (Ku_t - Kd_t).D_{t-1}.T} \quad \text{(B1)}$$

$$E_{t-1} = \frac{CFE_t}{Ke_t} \quad \text{(B2)}$$

$$Ke_t = \frac{Ku_t.E_{t-1}.Ke_t}{E_{t-1}.Ke_t - (Ku_t - Kd_t).D_{t-1} + (Ku_t - Kd_t).D_{t-1}.T} \quad \text{(B3) = (B2) in (A4d)}$$

$$\frac{Ku_t.E_{t-1}}{E_{t-1}.Ke_t - (Ku_t - Kd_t).D_{t-1}.(1 - T)} = 1 \quad \text{(B4a)}$$

$$E_{t-1}.Ke_t - (Ku_t - Kd_t).D_{t-1}.(1 - T) = Ku_t.E_{t-1} \quad \text{(B4b)}$$

$$Ke_t = \frac{Ku_t.E_{t-1} + (Ku_t - Kd_t).D_{t-1}.(1 - T)}{E_{t-1}} \quad \text{(B4c)}$$

$$\mathbf{Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}}.(1 - T)} \quad \textbf{(B4d)}$$

2. **Particular Case for Ke if g=0, ψ_t=Ku_t and V^{TS}_{t-1}=(D_{t-1}.Kd_t.T)/Ku_t:**

$$Ke_t = \frac{Ku_t.CFE_t - g.\left[(Ku_t - Kd_t).D_{t-1} - \left(Ku_t - \psi_t\right).V_{t-1}^{TS}\right]}{CFE_t - (Ku_t - Kd_t).D_{t-1} + \left(Ku_t - \psi_t\right).V_{t-1}^{TS}} \quad \text{(A4d)}$$

$$Ke_t = \frac{Ku_t.CFE_t - 0.\left[(Ku_t - Kd_t).D_{t-1} - (Ku_t - Ku_t).V_{t-1}^{TS}\right]}{CFE_t - (Ku_t - Kd_t).D_{t-1} + (Ku_t - Ku_t).V_{t-1}^{TS}} \quad \text{(B5)}$$

$$Ke_t = \frac{Ku_t.CFE_t}{CFE_t - (Ku_t - Kd_t).D_{t-1}} \quad \text{(B6)}$$

$$E_{t-1} = \frac{CFE_t}{Ke_t - 0} \quad \text{(B7)}$$

$$Ke_t = \frac{Ku_t.E_{t-1}.Ke_t}{E_{t-1}.Ke_t - (Ku_t - Kd_t).D_{t-1}} \quad \text{(B8) = (B7) in (B6)}$$

$$\frac{Ku_t.E_{t-1}}{E_{t-1}.Ke_t - (Ku_t - Kd_t).D_{t-1}} \quad \text{(B9a)}$$

$$Ku_t.E_{t-1} = E_{t-1}.Ke_t - (Ku_t - Kd_t).D_{t-1} \quad \text{(B9b)}$$

$$E_{t-1}.Ke_t = Ku_t.E_{t-1} + (Ku_t - Kd_t).D_{t-1} \quad \text{(B9c)}$$

$$\mathbf{Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}}} \quad \textbf{(B9d)}$$

3. Particular Case for WACC if $g>=0$, $\psi_t=Ku_t$, and $V^{TS}_{t-1}=(D_{t-1}.Kd_t.T)/Ku_t$:

$$WACC_t = \frac{Ku_t.FCF_t + g.[(Ku_t - \psi_t).V^{TS}_{t-1} + TS_t]}{FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t} \quad (A10d)$$

$$WACC_t = \frac{Ku_t.FCF_t + g.[(Ku_t - Ku_t).V^{TS}_{t-1} + TS_t]}{FCF_t + (Ku_t - Ku_t).V^{TS}_{t-1} + TS_t} \quad (B10)$$

$$E_{t-1} + D_{t-1} = \frac{FCF_t}{WACC_t - g} \quad (A8)$$

$$E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t}{Ku_t - g} \quad (A12d)$$

$$FCF_t + (Ku_t - Ku_t).V^{TS}_{t-1} + TS_t = (E_{t-1} + D_{t-1}).(Ku_t - g) \quad (B11)$$

$$WACC_t = \frac{Ku_t.(E_{t-1} + D_{t-1}).(WACC_t - g) + g.TS_t}{(E_{t-1} + D_{t-1}).(Ku_t - g)} \quad (B23) = (A8), (B11) \text{ in } (B10)$$

$$WACC_t.(E_{t-1} + D_{t-1}).(Ku_t - g) = Ku_t.(E_{t-1} + D_{t-1}).(WACC_t - g) + g.TS_t \quad (B12a)$$

$$WACC_t.(E_{t-1} + D_{t-1}).(Ku_t - g) = Ku_t.(E_{t-1} + D_{t-1}).WACC_t - g.Ku_t.(E_{t-1} + D_{t-1}) + g.TS_t \quad (B12b)$$

$$WACC_t.[(E_{t-1} + D_{t-1}).(Ku_t - g) - Ku_t.(E_{t-1} + D_{t-1})] = g.[TS_t - Ku_t.(E_{t-1} + D_{t-1})] \quad (B12c)$$

$$WACC_t.[(E_{t-1} + D_{t-1}).(Ku_t - g - Ku_t)] = g.[TS_t - Ku_t.(E_{t-1} + D_{t-1})] \quad (B12d)$$

$$WACC_t.(E_{t-1} + D_{t-1}).g = g.[Ku_t.(E_{t-1} + D_{t-1}) - TS_t] \quad (B12e)$$

$$WACC_t = \frac{Ku_t.(E_{t-1} + D_{t-1}) - TS_t}{E_{t-1} + D_{t-1}} \quad (B12f)$$

$$\mathbf{WACC_t = Ku_t - \frac{TS_t}{E_{t-1} + D_{t-1}}} \quad (\mathbf{B12g})$$

$$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - (Ku_t - \psi_t).\frac{V^{TS}_{t-1}}{E_{t-1}} \quad (A1)$$

$$Ke_t = Ku_t + (Ku_t - Kd_t).\frac{D_{t-1}}{E_{t-1}} - (Ku_t - Ku_t).\frac{V^{TS}_{t-1}}{E_{t-1}} \quad (B13a)$$

$$Ke_t = Ku_t + Ku_t.\frac{D_{t-1}}{E_{t-1}} - Kd_t.\frac{D_{t-1}}{E_{t-1}} \quad (B13b)$$

$$Ku_t.\left(\frac{E_{t-1} + D_{t-1}}{E_{t-1}}\right) = \frac{Ke_t.E_{t-1} + Kd_t.D_{t-1}}{E_{t-1}} \quad (B13c)$$

$$\mathrm{Ku_t = \frac{Ke_t.E_{t-1} + Kd_t.D_{t-1}}{E_{t-1} + D_{t-1}}} \qquad \text{(B13d)}$$

$$\mathrm{WACC_t = \frac{Ke_t.E_{t-1} + Kd_t.D_{t-1}}{E_{t-1} + D_{t-1}} - \frac{TS_t}{E_{t-1} + D_{t-1}}} \qquad \text{(B14) = (B13d) in (B12g)}$$

$$\mathrm{WACC_t = \frac{Ke_t.E_{t-1} + Kd_t.D_{t-1}}{E_{t-1} + D_{t-1}} - \frac{Kd_t.D_{t-1}.T}{E_{t-1} + D_{t-1}}} \qquad \text{(B15a)}$$

$$\mathbf{WACC_t = \frac{Ke_t.E_{t-1} + Kd_t.D_{t-1}.(1 - T)}{E_{t-1} + D_{t-1}}} \qquad \textbf{(B15b)}$$

Appendix C

1. **Value of the Levered Firm as a Non-growing Perpetuity according to Modigliani & Miller (1963) and Myers (1974), which corresponds to the particular case when g=0 and ψ=Kd.**

$$\mathbf{E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t}{Ku_t - g}} \qquad \textbf{(A12)}$$

$$\mathrm{V^{TS}_{t-1} = \frac{TS_t}{\psi_t - g}} \qquad \text{(C1)}$$

$$\mathrm{TS_t = D_{t-1}.Kd_t.T} \qquad \text{(C2)}$$

$$\mathrm{V^{TS}_{t-1} = \frac{D_{t-1}.Kd_t.T}{Kd_t - 0} = D_{t-1}.T} \qquad \text{(C3) = (C2) in (C1)}$$

$$\mathrm{V^{u}_{t-1} = \frac{FCF_t}{Ku_t - g}} \qquad \text{(C4)}$$

$$\mathrm{E_{t-1} + D_{t-1} = \frac{FCF_t}{Ku_t - g} + \frac{(Ku_t - \psi_t).V^{TS}_{t-1} + TS_t}{Ku_t - g}} \qquad \text{(C5)}$$

$$\mathrm{E_{t-1} + D_{t-1} = V^{u}_{t-1} + \frac{(Ku_t - Kd_t).D_{t-1}.T + D_{t-1}.Kd_t.T}{Ku_t - 0}} \qquad \text{(C6) = (C2), (C3) and (C4) in (C5)}$$

$$\mathrm{E_{t-1} + D_{t-1} = V^{u}_{t-1} + \frac{D_{t-1}.T.(Ku_t - Kd_t + Kd_t)}{Ku_t}} \qquad \text{(C7a)}$$

$$\mathrm{E_{t-1} + D_{t-1} = V^{u}_{t-1} + \frac{D_{t-1}.T.(Ku_t)}{Ku_t}} \qquad \text{(C7b)}$$

$$\mathbf{E_{t-1} + D_{t-1} = V^{u}_{t-1} + D_{t-1}.T} \qquad \textbf{(C7c)}$$

2. **Value of the Levered Firm as a Growing Perpetuity according to Modigliani & Miller (1963) and Lewllen and Emery, 1986 which corresponds to the particular case when g>0 and ψ=Kd.**

$$\mathbf{E_{t-1} + D_{t-1} = \frac{FCF_t + (Ku_t - \psi_t).V^{TS}_{t-1} + TS_t}{Ku_t - g}} \qquad (\mathbf{A12})$$

$$E_{t-1} + D_{t-1} = \frac{FCF_t}{Ku_t - g} + \frac{\left(Ku_t - \psi_t\right).V^{TS}_{t-1} + TS_t}{Ku_t - g} \qquad (C8a)$$

$$E_{t-1} + D_{t-1} = V^{u}_{t-1} + \frac{(Ku_t - Kd_t).D_{t-1}.T + D_{t-1}.Kd_t.T}{Ku_t - g} \qquad (C8b)$$

$$E_{t-1} + D_{t-1} = V^{u}_{t-1} + \frac{D_{t-1}.T.(Ku_t - Kd_t + Kd_t)}{Ku_t - g} \qquad (C9a)$$

$$\mathbf{E_{t-1} + D_{t-1} = V^{u}_{t-1} + \frac{D_{t-1}.T.Ku_t}{Ku_t - g}} \qquad (\mathbf{C9b})$$

Appendix D

Formula for D% as a Function of Growth Rate g

$$D\% = \frac{D_{t-1}}{D_{t-1} + E_{t-1}} \qquad (D1)$$

$$D\%.(D_{t-1} + E_{t-1}) = D_{t-1} \qquad (D2a)$$

$$D\%.D_{t-1} + D\%.E_{t-1} = D_{t-1} \qquad (D2b)$$

$$D\%.E_{t-1} = D_{t-1}.(1 - D\%) \qquad (D2c)$$

$$E_{t-1} = \frac{D_{t-1}.(1 - D\%)}{D\%} \qquad (D2d)$$

$$E_{t-1} + D_{t-1} = \frac{D_{t-1}.(1 - D\%)}{D\%} + D_{t-1} \qquad (D3)$$

$$E_{t-1} + D_{t-1} = \frac{D_{t-1}.(1 - D\%) + D_{t-1}.D\%}{D\%} \qquad (D4a)$$

$$E_{t-1} + D_{t-1} = \frac{D_{t-1} - D_{t-1}.D\% + D_{t-1}.D\%}{D\%} \qquad (D4b)$$

$$E_{t-1} + D_{t-1} = \frac{D_{t-1}}{D\%} \qquad (D4c)$$

$$E_{t-1} + D_{t-1} = \frac{FCF_t}{Ku_t - g} + V^{TS}_{t-1} \qquad \text{(D5)}$$

$$V^{TS}_{t-1} = \frac{TS_t}{\psi_t - g} \qquad \text{(D6)}$$

$$E_{t-1} + D_{t-1} = \frac{FCF_t}{Ku_t - g} + \frac{TS_t}{\psi_t - g} \qquad \text{(D7) = (D6) in (D5)}$$

$$TS_t = D_{t-1}.Kd_t.T \qquad \text{(D8)}$$

$$\frac{D_{t-1}}{D\%} = \frac{FCF_t}{Ku_t - g} + \frac{D_{t-1}.Kd_t.T}{\psi_t - g} \qquad \text{(D9) = (D4c), (D8) in (D7)}$$

$$\frac{D_{t-1}}{D\%} = \frac{FCF_t.\left(\psi_t - g\right) + D_{t-1}.Kd_t.T.(Ku_t - g)}{(Ku_t - g).(\psi_t - g)} \qquad \text{(D10)}$$

$$\mathbf{D\%} = \frac{\mathbf{D_{t-1}.(Ku_t - g).(\psi_t - g)}}{\mathbf{FCF_t.\left(\psi_t - g\right) + D_{t-1}.Kd_t.T.(Ku_t - g)}} \qquad (\mathbf{D11})$$